Biomedical Technology
About this Series

This series aims to report new developments in applied and computational mechanics—quickly, informally and at a high level. This includes the fields of fluid, solid and structural mechanics, dynamics and control, and related disciplines. The applied methods can be of analytical, numerical and computational nature.

More information about this series at http://www.springer.com/series/4623
One of the current challenges in medicine and engineering is related to the application of computational methods to clinical medicine. The virtual environment can be used to study biological systems at different scales and under multi-physics conditions. Based on the tremendous advances in medical imaging, modern CAD systems, and high-performance computing, engineering can provide help in understanding biological processes but also implant designs. This enables the possibility to enhance medical decision processes in many areas of clinical medicine. The computational tools and methods can be applied to predict performance of medical devices in virtual patients. Physical and animal testing procedures can be reduced by use of virtual prototyping of medical devices.

In this book, scientists from different areas of medicine, engineering, and natural sciences have contributed to the above research areas and ideas. The book will focus on function, production, initialization, and complications of different types or implants and related topics.

The contributions start with theoretical and numerical investigations that are related to modeling biological materials like the papers “RVE Procedure for Estimating the Elastic Properties of Inhomogeneous Microstructures Such as Bone Tissue” by Blöß and Welsch and “A Gradient-Enhanced Continuum Damage Model for Residually Stressed Fibre-Reinforced Materials at Finite Strains” by Waffenschmidt et al. A more application-oriented work “A Mechanically Stimulated Fracture Healing Model Using a Finite Element Framework” is provided by Sapotnick and Nackenhorst that builds a bridge to the work “The Customized Artificial Hip Cup: Design and Manufacturing of an Innovative Prosthesis” by Betancur Escobar et al. New stents are modeled in the paper “On the Role of Phase Change in Modelling Drug-Eluting Stents” by Bozsak et al. The paper “Development of Magnesium Alloy Scaffolds to Support Biological Myocardial Grafts: A Finite Element Investigation” by Weidling et al. deals with the development of new degenerative implants. The contributions “Finite Element Analysis of Transcatheter Aortic Valve Implantation in the Presence of Aortic Leaflet Calcifications” by Dimasi et al., “Repair of Mitral Valve Prolapse Through ePTFE Neochordae: A Finite Element Approach From CMR” by Sturla et al. and
“An Extended Computational Framework to Study Arterial Vasomotion and Its Links to Vascular Disease” by Boileau et al. are related to virtual models for the vascular system. Models that describe the behavior of the cochlea are provided in “Development of a Model of the Electrically Stimulated Cochlea” by Nogueira et al. Finally, models and investigations of infections due to implantation are discussed in “Implant Related Infections” by Abraham and “Animal Test Models for Implant-Associated Inflammation and Infections” by Rais et al.

All contributions show the state of the art in modeling and numerical simulation of systems in biotechnology and thus provide an extensive overview of this subject.

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RVE Procedure for Estimating the Elastic Properties of Inhomogeneous Microstructures Such as Bone Tissue

Tanja Blöß and Michael Welsch

Abstract Cancellous bone can roughly be seen as a two-phase material consisting of the bone tissue reinforcement and the interstitial bone marrow matrix. Thus, for a computer-aided mechanical stress analysis of bones a constitutive law is required, which can predict the inhomogeneous elasticity depending on the local bone density and microstructure. Besides several measurement methods, the method of representative volume element (RVE) in combination with the finite element solution technique has been established for this purpose. This work investigates this method in detail. Therefore, random but statistical equivalent RVEs are created to have unlimited access to different structures. Generally, an apparent and not an effective stiffness is obtained due to the RVE method. However, a very close solution can be achieved if several issues are considered carefully. These issues can be divided into the set of boundary conditions, the RVE size and averaging the randomness. The influences are investigated accurately. A new approach is proposed to deduce an isotropic constitutive law from the anisotropic stiffness matrix. There are unlimited possible solutions in theory. However, the Voigt and Reuss approximations give the possible bounds. A method is described, which allows to obtain the effective stiffness by merging these bounds. A structural analysis is performed with different RVEs and the effective stiffness is estimated for varying parameters. An empirical equation is introduced, which covers the whole stiffness range. Therein, the microstructure is modelled with a single parameter. Real bone measurements can be fitted with this equation as well.

Keywords Cancellous bone · Boundary conditions · Elastic properties · FEM · Homogenization · Voigt and Reuss approximation
1 Introduction

Computer simulations become more and more important for endoprosthetic investigations of bones. Therefore, a realistic material modeling is required to ensure a reliable prediction of the inner mechanical stresses. Bones generally consist of cancellous bone surrounded by a thin layer of dense compact bone resulting in location-dependent material properties. The modeling of the microstructure in detail is computational out of reach nowadays. A pointwise homogenization of the stochastic and heterogeneous microstructure would be beneficial. Thus, a constitutive law is required that can predict the inhomogeneous elasticity depending on the local bone density and microstructure.

Direct mechanical measurements for example are performed by Ashmann et al. [1, 2], Rho et al. [3, 4], Dalstra et al. [5] and different regression equations are proposed. In the mid nineties a new idea was investigated. Real microstructures based on high resolution CT images are converted into virtual models that could be studied by FEM simulation. Such studies are performed by Müller [6], Ulrich [7], Pahr and Zysset [8]. Different issues raise by dealing with the continuum mechanics approach. Ulrich et al. investigated the influence of meshing and element formulation. Pahr and Zysset compared several sets of boundary conditions regarding the accuracy of the obtained stiffness of human cancellous bone specimens.

Since the procedure of calculating the anisotropic stiffness matrix seems to be clear, it lacks of estimating a corresponding isotropic constitutive law. The theory of micromechanics and homogenization points out to distinguish between apparent and effective estimates. As a general rule, an apparent estimate is obtained since the window size is limited. However, a convergence study allows the prediction of an effective estimate by increasing the window size stepwise (cp. Kanit [9, 10]).

Notwithstanding that the FEM solution is an approximation by nature, an apparent estimate should be expected generally due to use of boundary conditions.

This work presents a study of the different influences and proposes a procedure to calculate effective moduli. Methods are presented to determine the “effectiveness” of the solution. Plenty different, but stochastically equivalent structures are needed to study the influences entirely. An algorithm is applied to generate an unlimited number of varying representative volume elements (RVE).

2 Material and Method

2.1 Generation of Stochastic RVE

Cancellous bone can roughly be seen as a two-phase material consisting of the bone tissue reinforcement and the interstitial bone marrow matrix. Generally, the effective elastic properties of such materials are depending on the respective volume fractions, the elastic properties of both materials and the structural composition.
A simple algorithm generates random RVE structures in three steps. First an initial number of cells are randomly assigned with material within a three-dimensional grid. Afterwards additional cells are randomly selected, but only assigned with material, if they are adjacent to existing material. This is done until a given volume fraction is reached. In a final step, all remaining grid cells are assigned with the matrix material. This algorithm is illustrated in Fig. 1.

While the volume fraction is directly regulated, the RVE structure develops indirectly by the number of initial cells. A rather rough cluster with high material agglomerations emerges from a small number of initial cells, whereas a fine dispersed cluster emerges from many initial cells. Figure 2 shows three different RVEs, each with 50 elements per edge and equal volume fraction but varying number of initial cells.

### 2.2 Continuum Mechanics Approach

The continuum mechanics approach is used to calculate three-dimensional material deformations. For static considerations the momentum balance of the current
configuration is reduced to a time and mass invariant equilibrium that can be expressed by the divergence of the Cauchy stress tensor.

\[ \text{div} \sigma = 0 \]  

(1)

This expression is under-constrained and additional definitions are required. First of all, the continuity of the field quantities is postulated, meaning that all deformations are physically objective and infinitesimal small material particles are not allowed to penetrate each other or fluctuate. This uniqueness is obtained by the definition of the deformation tensor.

\[ F = \frac{dx}{dX} \]  

(2)

In terms of the physical objectivity, the material behavior of elastic bodies (also denoted as Cauchy elasticity) now demand tensor compatibility of stress and deformation.

\[ \sigma = f(F) \]  

(3)

Such compatibility is given by the Rivlin-Ericksen theorem for isotropic behavior [11].

\[ \sigma = a_0 + a_1 F^T F + a_2 (F^T F)^2 \]  

(4)

Thereby, the coefficients \( a_0, a_1 \) and \( a_2 \) are arbitrary scalar functions of the invariants of the deformation tensor. Usually, this dependence is formulated in relation to the strain energy density \( \psi \). Thus, the Green or hyper-elastic material behavior can be defined by a pure scalar function \( \psi = f(I_1, I_2, J) \), with the constitutive law:

\[ \sigma = J^{-1} \frac{d\psi}{dF} F^T \]  

(5)

However, for this assumption isotropy is presumed, meaning that not every material can be modeled by this constitutive law. In order to consider anisotropic effects as well, the constitutive law is expressed by using the geometric linearized form of the right Cauchy-Green tensor.

\[ \varepsilon = \frac{1}{2} (F^T F - I) \]  

(6)

This modified tensor can be used to derive a simplified relation between stress and deformation. In analogy to (4) an equalization of \( f(F) \sim f(\varepsilon) \) leads to Hooke’s law in continuum mechanics

\[ \sigma = \frac{1}{2} (\lambda \text{tr} \varepsilon) + 2G\varepsilon \]  

(7)
with the assumption \( \sigma(\varepsilon = 0) = 0 \) and the Lame’s constants \( \lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} \) and \( G = \frac{E}{2(1+\nu)} \) [12].

The physical validity of this equation expires with increasing deformation due to the linearization and does not describe the stress decrease of real material. However, in practical (5) and (7) only differ for deformations higher than 5% strain.

The vectorization of the stress and strain tensors by use of the Voigt notation leads to the well-known matrix equation:

\[
\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33} \\
\sigma_{12} \\
\sigma_{13} \\
\sigma_{23}
\end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix}
1 - \nu & \nu & 0 & 0 & 0 \\
\nu & 1 - \nu & \nu & 0 & 0 \\
\nu & \nu & 1 - \nu & 0 & 0 \\
0 & 0 & 0 & \frac{1-2\nu}{2} & 0 \\
0 & 0 & 0 & 0 & \frac{1-2\nu}{2} \\
0 & 0 & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
\varepsilon_{11} \\
\varepsilon_{22} \\
\varepsilon_{33} \\
\varepsilon_{12} \\
\varepsilon_{13} \\
\varepsilon_{23}
\end{bmatrix}
\] (8)

This linearized equation still describes pure isotropic material behavior. It can be converted into the generalized Hooke’s law by a phenomenologically motivated consideration, so that in principle all 36 coefficients can be chosen independently.

\[
\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33} \\
\sigma_{12} \\
\sigma_{13} \\
\sigma_{23}
\end{bmatrix} = \begin{bmatrix}
C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\
C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\
C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\
C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\
C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\
C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66}
\end{bmatrix} \begin{bmatrix}
\varepsilon_{11} \\
\varepsilon_{22} \\
\varepsilon_{33} \\
\varepsilon_{12} \\
\varepsilon_{13} \\
\varepsilon_{23}
\end{bmatrix}
\] (9)

However, both the strain tensor \( \varepsilon \) and the stress tensor \( \sigma \) are symmetric, so the stiffness matrix \( C \) and the respective compliance matrix \( N = C^{-1} \) have to be symmetric as well. Consequently, only 21 independent coefficients remain, which have to provide a positive determinant.

One should consider that this kind of anisotropic modeling is only valid for homogeneous bodies. However, anisotropic behavior in general is caused by inhomogeneity, so this is a crude assumption with very limited validity. For example, a structure causing momentums cannot be homogenized with the constitutive law (9).

However, the estimated stiffness is strongly influenced by the numerical process even for suitable structures.

### 2.3 Homogenization Approach

In general an analogical homogeneous constitutive law for the underlying inhomogeneous microstructure should fulfill the Hill condition [13].

\[
<\sigma> <\varepsilon> = <\sigma \varepsilon>
\] (10)