RANDOMNESS AND UNDECIDABILITY IN PHYSICS

K Svozil

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Karl Svozil
Institute for Theoretical Physics
Technical University of Vienna
Austria
Dedicated to the memory of my father

Karl Svozil

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Preface

Recent findings in the computer sciences, discrete mathematics, formal logics and metamathematics have opened up a *via regia* for the investigation of undecidability and randomness in physics. A translation of these formal and abstract concepts yields a fresh look into diverse features of physical modelling such as quantum complementarity and the measurement problem, but also stipulates questions related to the necessity of the assumption of continua.

Any physical system may be perceived as a computational process. One could even speculate that physical systems *exactly* correspond to, and indeed are, computations of a very specific kind; with a particular computer model in mind. From this point of view it is absolutely reasonable to investigate physical systems with concepts and methods developed by the computer sciences.

Conversely, any computer may be perceived as a physical system; not only in the immediate sense of the physical properties of its hardware. Computers are a medium to virtual realities. The foreseeable importance of such virtual realities stimulates the investigation of an "inner description," a "virtual physics," if you like, of these universes of computation. Indeed, one may consider our own universe as just one particular realisation of an enormous number of virtual realities, most of them awaiting discovery.

Besides these issues, the intuitive terms "rational" (human thought), "conceivable" and so on, have been made precise by the concepts of mechanic computation and recursive enumeration. The reader may find these developments sufficiently exciting to go on and study this new field.

The first part of this book introduces the fundamental concepts. Informally stated, recursive function theory is concerned with the question of whether an entity is computable in a very precisely defined way. Algorithmic information theory deals with the quantitative description of computation, in particular with the shortest program length. Coding and suitable algebraic representation of physical statements are the prerequisites for their algorithmic treatment.

One motive of this book is the recognition that what is often referred to as "randomness" in physics might actually be a signature of undecidability for systems whose evolution is computable on a step-by-step basis. Therefore the second part of the book is devoted to the investigation of undecidability.

To give a flavour of the type of questions envisaged: Consider an arbitrary algorithmic system which is computable on a step-by-step basis. — Any computer program is such a system. It is in general impossible to specify another algorithmic procedure (including itself) which, by performing experiments and successive input/output analysis on the first system, finds the deterministic law by which it is governed. But even if such a law is specified, it is in general impossible to predict the system behaviour in the "distant
future.' In other words: no "speedup" or "computational shortcut" is possible. These statements are consequences of two classical theorems in recursion theory, the recursive unsolvability of the rule inference problem and of the halting problem.

Certain self-referential statements like "I am lying" are paradoxical and resemble the absurd attempt of Freiherr von Münchhausen to rescue himself from a swamp by dragging himself out by his own hair. Such paradoxes can only be consistently avoided by accepting restrictions to the expressive power and to the comprehension of the associated methods and systems — with undecidability and incompleteness as consequence.

Complementarity is a feature which can be modelled by experiments on certain finite automata. This is due to the fact that measurement of one observable of the automaton destroys the possibility to measure another observable of the same automaton and vice versa. Certain self-referential measurements pursue a similar attempt: on the one hand they pretend to render the "true" value of an observable, while on the other hand they have to interact with the object to be measured and thereby inevitably change its state.

It is important to distinguish between the "intrinsic" view of an observer, who is entangled with and who is an inseparable part of the system, and the "extrinsic" perspective of an observer who is not entangled with the system via self-reference. Indeed, the recognition of the importance of intrinsic perception, of a "view from within," might be considered as a key observation towards a better understanding of undecidability and complementarity.

The third, last part of the book is dedicated to a formal definition of randomness and entropy measures based on algorithmic information theory.

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Karl Svozil

Institut für Theoretische Physik
Technische Universität Wien
Wiedner Hauptstraße 8-10/136
A-1040 Vienna, Austria

e1360dab@awiuni11.bitnet
e1360dab@AWIUNI11.EDVZ.UniVie.AC.AT
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