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Stability and Optimization of Structures
Generalized Sensitivity Analysis
Mechanical Engineering Series

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Series Preface

Mechanical engineering, and engineering discipline born of the needs of the industrial revolution, is once again asked to do its substantial share in the call for industrial renewal. The general call is urgent as we face profound issues of productivity and competitiveness that require engineering solutions, among others. The Mechanical Engineering Series is a series featuring graduate texts and research monographs intended to address the need for information in contemporary areas of mechanical engineering.

The series is conceived as a comprehensive one that covers a broad range of concentrations important to mechanical engineering graduate education and research. We are fortunate to have a distinguished roster of series editors, each an expert in one of the areas of concentration. The names of the series editors are listed on page v of this volume. The areas of concentration are applied mechanics, biomechanics, computational mechanics, dynamic systems and control, energetics, mechanics of materials, processing, thermal science, and tribology.
In our modern world, best structures with specified shape, stiffness, strength, stability, frequency, and so on, can be designed with the assistance of computer-aided methodologies including sensitivity analysis, reliability-based design, inverse engineering, optimization, and anti-optimization. Buckling is an extremely important design constraint for structures with slender members such as latticed domes and frames; buckling of geometrically nonlinear structures is a well developed field of research. Nevertheless, because of the complexity of nonlinear buckling behavior, optimization of nonlinear structures has come to be conducted only recently despite its importance.

It is possible to consider structural optimization as a straightforward application of mathematical programming and operations research, as well as heuristics and evolutionary approaches. Such, however, is not the case for optimization of structures that undergo buckling. As cautioned by the “danger of naive optimization [287],” optimized structures often become more imperfection-sensitive and their buckling loads are reduced sharply because of the inevitable presence of initial imperfections that arise from errors in manufacturing processes, material defects, and other causes. It is certainly ironic that dangerous structures are produced in the attempt to optimize their performance. In any search for the best structure, the imperfection sensitivity of optimized structures must be investigated.

This book offers an introduction to “optimization of geometrically nonlinear structures under stability constraint,” which is an exciting and fast-growing branch of application of structural and mechanical engineering, and also necessarily involves applied mathematics. The premise of this book is that a thorough and profound knowledge of nonlinear buckling behaviors is crucial, via proper problem setting, as a step toward the successful design of the best structure.
Some optimized structures are shown to be safe, and readers are encouraged to carry out optimization-based design with confidence.

In Part I, design sensitivity analysis and imperfection sensitivity analysis are introduced as systematic tools to perform stability design of structures. The influence of design parameters on structural performance is to be expressed as parameter sensitivity. Design sensitivity analysis is implemented into the gradient-based algorithm for structural optimization in Part II. Imperfection sensitivity laws are introduced to evaluate the influence of initial imperfections on buckling loads quantitatively. In this book, design sensitivity analysis and imperfection sensitivity analysis, which have been addressed independently up to now, are described in a synthetic manner based on the general theory of elastic stability [166]. This theory, which once was an established means to describe the buckling of structures, is thus given a new role in the computer age. Part I is organized as follows.

- The overview of design sensitivity analysis and its theoretical backgrounds are presented in Chapter 1.
- Numerical methods of design sensitivity analysis are provided in Chapter 2.
- Imperfection sensitivity analysis is presented in the framework of modern stability theory in Chapter 3.

In Part II, based on the synthetic description of sensitivity analyses presented in Part I, we introduce state-of-the-art optimization methodologies of geometrically nonlinear finite-dimensional structures under stability constraints. These optimization methodologies are reinforced on the one hand by the stability theory and on the other hand by finite element method and mathematical programming with ever-increasing computing power. Design of compliant mechanisms is highlighted as an engineering application of shape and topology optimization with extensive utilization of snapthrough buckling. Part II is organized as follows.

- In Chapter 4, general formulation for optimization under stability constraints is provided. An optimized truss dome is shown to be less imperfection-sensitive than a non-optimal one.
- Optimal structures with snapthrough are investigated in Chapter 5 to pave the way for shape design of compliant mechanisms using snapthrough behavior in Chapter 6.
- Optimal frames with coincident buckling loads are investigated in Chapter 7.
- Imperfection sensitivity of hilltop branching points with simple, multiple, and degenerate bifurcation points are investigated in Chapters 8–10.

In Part III, in order to ensure the performance of optimized structures, we introduce two design methodologies:

- optimization via the worst imperfection, and
- probabilistic analysis via random imperfections.
In particular, imperfection sensitivity laws are extended to be applicable to many imperfection variables and, in turn, to deal with the probabilistic variation of the buckling loads of structures. Part III is organized as follows.

- The asymptotic theory on the worst imperfection is formulated in Chapter 11.
- An anti-optimization problem is formulated in Chapter 12 to minimize the lowest eigenvalue of the tangent stiffness matrix, and a design methodology is presented for a laterally braced frame.
- The worst imperfection is defined and investigated for a stable-symmetric bifurcation point in Chapter 13.
- The theory on random imperfections is presented in Chapter 14, and is applied to steel specimens with hilltop branching in Chapter 15.
- The theory is extended to the second-order imperfections in Chapter 16.

In the Appendix, derivations of several formulations and details of numerical examples are presented. In particular, the derivation of imperfection sensitivity laws by the power series expansion method is an important ingredient for readers who are interested in stability theory.

This book consequently offers a wide and profound insight into optimization-based and computer-assisted stability design of finite-dimensional structures in a readable and illustrative form for graduate students of engineering and applied mathematicians. General methodology is emphasized instead of studies of particular structures. Historical developments are outlined with many references to assist readers’ further studies.

The authors are grateful to Dr. J. S. Arora for his support of an optimization program. The suggestion of Dr. K. K. Choi was vital for the publication of this book. The authors thank for the comments of Drs. K. Murota and Y. Kanno. For the realization of this book, the authors owe much to Drs. K. Uetani, K. Terada, S. Okazawa, S. Nishiwaki, J. Takagi, K. Oide, and Mr. J. Y. Zhang. The support of C. Simpson, E. Tham and K. Stanne was indispensable for the publication of this book. The authors conclude the preface with many thanks to Dr. I. Elishakoff for his encouragement.

February 2007

Makoto Ohsaki
Kiyohiro Ikeda
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