Modelling and Applications in Mathematics Education
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Modelling and Applications in Mathematics Education

The 14th ICMI Study

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Among the themes that have been central to mathematics education during the last 30 years are those of mathematical *modelling* and *applications* of mathematics to extra-mathematical fields. More generally we refer to these as *relations between mathematics and the extra-mathematical world* (sometimes also called the "real world") or preferably, according to Henry Pollak, the "rest of the world". That applications and modelling have been important themes in mathematics education can be inferred from the wealth of literature on these topics, including material generated from a multitude of national and international *conferences*. In particular let us mention firstly the ICMEs (the International Congresses on Mathematical Education), with their regular working or topic groups and lectures on applications and modelling; and secondly the series of ICTMAs (the International Conferences on the Teaching of Mathematical Modelling and Applications) which have been held biennially since 1983. Their Proceedings and Survey Lectures, have addressed the state-of-the-art at the relevant time, and contain many examples, studies, conceptual contributions and resources involving relations between the real world and mathematics, for all levels of the educational system. In *curricula and textbooks* we find today many more references to real world phenomena and problems than, say, twenty years ago. Yet while applications and modelling play more important roles in many countries' classrooms than in the past, there still exists a substantial gap between the ideals expressed in educational debate and innovative curricula on the one hand, and everyday teaching practice on the other. In particular, genuine modelling activities are still rather rare in mathematics classrooms.

Altogether, during the last few decades there has been considerable work in mathematics education that has centred on applications and modelling. Many activities have had a primary focus on practice, e.g. construction and trial of mathematical modelling examples for teaching and examination purposes, writing of application-oriented textbooks, implementation of applications and modelling in existing curricula, or development of innovative, modelling-oriented curricula. Several of these activities also contain research components such as: clarification of relevant concepts; investigation of competencies and identification of difficulties and strategies activated by students when dealing with application problems; observation and analysis of teaching; study of learning and communication processes in modelling-oriented lessons; and evaluation of alternative approaches used to assess performance in applications and modelling. In particular during the last ten
years the number of genuine research contributions has increased considera-

That applications and modelling have been, and continue to be, central
themes in mathematics education is not at all surprising. Nearly all questions
and problems in mathematics education, that is questions and problems con-
cerning human learning and the teaching of mathematics, influence and are
influenced by relations between mathematics and some aspects of the real
world. For instance, one essential answer (albeit not the only one) to the
question as to why all persons ought to learn mathematics is that it provides
a means for understanding the world around us, for coping with everyday
problems, or for preparing for future professions. When addressing the ques-
tion of how individuals acquire mathematical knowledge, we cannot avoid
the role of its relationship to reality, especially the relevance of situated
learning (including the problem of the dependence of learning on specific
contexts). General questions as to what “mathematics” is, as a part of our
culture and as a social phenomenon, of how mathematics has emerged and
developed, involve also “applications” of mathematics in other disciplines,
in nature and society. Today mathematical models and modelling have pene-
trated a great variety of disciplines, leaving only a few fields (if any) where
mathematical models do not play some role. This increasing involvement has
been substantially supported and accelerated by the availability of powerful
electronic tools, such as calculators and computers, with their enormous
communication capabilities.

Relations between the real world and mathematics are particularly rele-
vant within the current OECD (Organisation for Economic Co-operation and
Development) PISA project. What is being tested in PISA (Programme for
International Student Assessment), is mathematical literacy, that is, accord-
ing to the PISA framework, “an individual’s capacity to identify and under-
stand the role that mathematics plays in the world, to make well-founded
judgements and to use and engage in mathematics, in ways that meet the
needs of that individual’s life as a constructive, concerned, and reflective
citizen.” That means the emphasis in PISA is on the use of mathematical
knowledge in a multitude of situations and contexts. In several countries, this
project has initiated an intense discussion about aims and design of mathemat-
ics instruction in schools, and especially about the role of mathematical
modelling, applications of mathematics and relations to the real world. Such
deliberations are also occurring in countries outside the OECD.

This book is the Study Volume of ICMI Study 14 on “Applications and
Modelling in Mathematics Education”, which began effectively in 2002 with
the development of the Discussion Document by the Programme Committee
(published in Educational Studies in Mathematics 51(2002)1/2, pp 149-171). In
mounting this Study, ICMI has taken into account the reasons mentioned
above for the importance of relationships between mathematics and the real world, as well as the contemporary state of the educational debate, and of research and development in this field. This does not, of course, mean that we claim to know satisfactory answers to the essential questions in this area, and that the role of the Study is simply to provide a forum for putting these together. Rather, an important aim of the Study and this Volume has been to identify shortcomings, as well as to stimulate further research and development activities, in addition to reporting on existing research and practice.

Documenting the state-of-the-art in a field and identifying deficiencies and needed research requires a structuring framework. This is particularly important in an area which is as complex and difficult to survey as the teaching and learning of mathematical modelling and applications. As we have seen, this topic not only deals with most of the essential aspects of the teaching and learning of mathematics at large, it also touches upon a wide variety of versions of the real world outside mathematics that one seeks to model. Perceived in this way, the topic of applications and modelling may appear to encompass all of mathematics education plus much more. It is evident, therefore, that we need a way of conceptualising the field so as to reduce complexity to a meaningful and tractable level. That is why this Volume commences with an introductory Part I where we clarify some of the basic concepts and notions of the field, and offer a conceptualisation that helps to structure it and to identify important challenges and questions. This introductory part, at the same time, provides a concise access to the field for the uninitiated reader together with a brief sketch of its history.

Following from this introductory part, the Volume contains plenary papers given at the Study Conference (Dortmund, February 2004) and various papers that address important issues in the field. It is stressed, however, that this Study Volume is not simply the Proceedings of the Study Conference — rather, the production of this Volume has involved an independent process. Of course, the papers presented at the Study Conference provided a rich source for this Volume, and the majority of papers here were derived in some way from those Conference papers. However, many of the papers in this Volume have been produced independently of the Study Conference, in particular to fill gaps that became obvious during the Conference.

We would like to express our sincere thanks to the members of the Programme Committee for this Study who have contributed in various ways to producing this Volume. In particular, several members have acted as editors of Sections in this Volume. Without their work and devotion, this extensive Volume could not have been completed. Our thanks go equally to all the authors who have contributed to this Volume and thus helped to make it — so we hope — a rich source of information and inspiration for readers. We also thank ICMI very much for having given priority to this Study, and in par-
ticular its Secretary, Bernard Hodgson, for his sensitive way of channelling ICMI views and proposals into this Study while, at the same time, leaving the organisers and editors with all the freedom they wanted and needed to undertake this task. Eventually, we would like to thank the Publisher, Springer, also for their patience when the completion of this Volume was on their agenda.

Let us finish this Preface by expressing our hope that this ICMI Study 14 Volume will be of value both for mathematics educators, mathematics teachers and mathematicians as well as for interested professionals in other disciplines in which mathematics plays an essential role, and that it will contribute to a strengthening and further development of the field of applications and modelling in mathematics education, and to an intensification of various kinds of research and practice activities in the field.

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