Modeling and Optimization of LCD Optical Performance

Dmitry A. Yakovlev
Saratov State University, Russia

Vladimir G. Chigrinov
Hong Kong University of Science & Technology, Hong Kong

Hoi-Sing Kwok
Hong Kong University of Science & Technology, Hong Kong

Focussing on polarization matrix optics in many forms, this book includes coverage of a wide range of methods which have been applied to LCD modeling, ranging from the simple Jones matrix method to elaborate and high accuracy algorithms suitable for off-axis optics. Researchers and scientists are constantly striving for improved performance, faster response times, wide viewing angles, improved colour in liquid crystal display development, and with this comes the need to model LCD devices effectively.

The authors have significant experience in dealing with the problems related to the practical application of liquid crystals, in particular their optical performance.

Key features:
- Explores analytical solutions and approximations to important cases in the matrix treatment of different LCD layer configurations, and the application of these results to improve the computational method
- Provides the analysis of accuracies of the different approaches discussed in the book
- Explains the development of the eigenwave Jones matrix method which offers a path to improved accuracy compared to Jones matrix and extended Jones matrix formalisms, while achieving significant improvement in computational speed and versatility compared to full 4x4 matrix methods
- Includes a companion website hosting the authors’ program library

The website also contains a set of sample programs (source codes) using LMOPTCS (FORTRAN), a collection of routines for calculating the optical characteristics of stratified media, the use of which allows for the easy implementation of the methods described in this book.

Cover design: Cylinder

The Society for Information Display (SID) is an international society which has the aim of encouraging the development of all aspects of the field of information display. Complementary to the aims of the society, the Wiley–SID series is intended to explain the latest developments in information display technology at a professional level. The broad scope of the series addresses all facets of information displays from technical aspects through systems and prototypes to standards and ergonomics.

www.wiley.com/go/modelinglcd
MODELING AND OPTIMIZATION OF LCD OPTICAL PERFORMANCE
Wiley-SID Series in Display Technology

Series Editors:
Anthony C. Lowe and Ian Sage

Display Systems: Design and Applications
Lindsay W. MacDonald and Anthony C. Lowe (Eds.)

Electronic Display Measurement: Concepts, Techniques, and Instrumentation
Peter A. Keller

Reflective Liquid Crystal Displays
Shin-Tson Wu and Deng-Ke Yang

Colour Engineering: Achieving Device Independent Colour
Phil Green and Lindsay MacDonald (Eds.)

Display Interfaces: Fundamentals and Standards
Robert L. Myers

Digital Image Display: Algorithms and Implementation
Gheorghe Berbecel

Flexible Flat Panel Displays
Gregory Crawford (Ed.)

Polarization Engineering for LCD Projection
Michael G. Robinson, Jianmin Chen, and Gary D. Sharp

Introduction to Microdisplays
David Armitage, Ian Underwood, and Shin-Tson Wu

Mobile Displays: Technology and Applications
Achintya K. Bhowmik, Zili Li, and Philip Bos (Eds.)

Photoalignment of Liquid Crystalline Materials: Physics and Applications
Vladimir G. Chigrinov, Vladimir M. Kozenkov, and Hoi-Sing Kwok

Projection Displays, Second Edition
Matthew S. Brennesholtz and Edward H. Stupp

Introduction to Flat Panel Displays
Jiun-Haw Lee, David N. Liu, and Shin-Tson Wu

LCD Backlights
Shunsuke Kobayashi, Shigeo Mikoshiba, and Sungkyoo Lim (Eds.)

Ernst Lueder

Transreflective Liquid Crystal Displays
Zhijing Ge and Shin-Tson Wu

Liquid Crystal Displays: Fundamental Physics and Technology
Robert H. Chen

3D Displays
Ernst Lueder

OLED Display Fundamentals and Applications
Takatoshi Tsujimura

Illumination, Color and Imaging: Evaluation and Optimization of Visual Displays
Peter Bodrogi and Tran Quoc Khanh

Interactive Displays: Natural Human-Interface Technologies
Achintya K. Bhowmik (Ed.)

Addressing Techniques of Liquid Crystal Displays
Temkar N. Ruckmongathan

Fundamentals of Liquid Crystal Devices, Second Edition
Deng-Ke Yang and Shin-Tson Wu

Modeling and Optimization of LCD Optical Performance
Dmitry A. Yakovlev, Vladimir G. Chigrinov, and Hoi-Sing Kwok
MODELING AND OPTIMIZATION OF LCD OPTICAL PERFORMANCE

Dmitry A. Yakovlev  
*Saratov State University, Russia*

Vladimir G. Chigrinov  
*Hong Kong University of Science & Technology, Hong Kong*

Hoi-Sing Kwok  
*Hong Kong University of Science & Technology, Hong Kong*
To our beloved wives: Larisa, Larisa, and Ying-Hung
## Contents

Series Editor’s Foreword ............................................................... xiii
Preface ......................................................................................... xv
Acknowledgments ......................................................................... xix
List of Abbreviations ..................................................................... xxi
About the Companion Website .................................................... xxiii

1 Polarization of Monochromatic Waves. Background of the Jones Matrix Methods. The Jones Calculus ..................................................... 1

1.1 Homogeneous Waves in Isotropic Media .............................. 1
  1.1.1 Plane Waves ......................................................................... 1
  1.1.2 Polarization. Jones Vectors .................................................. 3
  1.1.3 Coordinate Transformation Rules for Jones Vectors, Orthogonal Polarizations. Decomposition of a Wave into Two Orthogonally Polarized Waves ......................................................... 9

1.2 Interface Optics for Isotropic Media ..................................... 14
  1.2.1 Fresnel’s Formulas. Snell’s Law ......................................... 14
  1.2.2 Reflection and Transmission Jones Matrices for a Plane Interface between Isotropic Media .................................................. 20

1.3 Wave Propagation in Anisotropic Media .............................. 23
  1.3.1 Wave Equations ................................................................. 23
  1.3.2 Waves in a Uniaxial Layer .................................................. 25
  1.3.3 A Simple Birefringent Layer and Its Principal Axes ............ 30
  1.3.4 Transmission Jones Matrices of a Simple Birefringent Layer at Normal Incidence ............................................................ 32
  1.3.5 Linear Retarders .................................................................. 36
  1.3.6 Jones Matrices of Absorptive Polarizers. Ideal Polarizer ...... 38

1.4 Jones Calculus ........................................................................ 41
  1.4.1 Basic Principles of the Jones Calculus ................................. 42
  1.4.2 Three Useful Theorems for Transmissive Systems .............. 46
  1.4.3 Reciprocity Relations. Jones’s Reversibility Theorem ........... 50
  1.4.4 Theorem of Polarization Reversibility for Systems Without Diattenuation .......................................................... 53
  1.4.5 Particular Variants of Application of the Jones Calculus. Cartesian Jones Vectors for Wave Fields in Anisotropic Media ......................... 55

References .................................................................................. 57
2 The Jones Calculus: Solutions for Ideal Twisted Structures and Their Applications in LCD Optics 59
  2.1 Jones Matrix and Eigenmodes of a Liquid Crystal Layer with an Ideal Twisted Structure 59
  2.2 LCD Optics and the Gooch–Tarry Formulas 64
  2.3 Interactive Simulation 67
  2.4 Parameter Space 69
  References 73

3 Optical Equivalence Theorem 75
  3.1 General Optical Equivalence Theorem 75
  3.2 Optical Equivalence for the Twisted Nematic Liquid Crystal Cell 77
  3.3 Polarization Conserving Modes 77
    3.3.1 LP1 Modes 78
    3.3.2 LP2 Modes 79
    3.3.3 LP3 Modes 80
    3.3.4 CP Modes 81
  3.4 Application to Nematic Bistable LCDs 82
    3.4.1 $2\pi$ Bistable TN Displays 82
    3.4.2 $\pi$ Bistable TN Displays 83
  3.5 Application to Reflective Displays 84
  3.6 Measurement of Characteristic Parameters of an LC Cell 86
    3.6.1 Characteristic Angle $\Omega$ 86
    3.6.2 Characteristic Phase $\Gamma$ 87
  References 87

4 Electro-optical Modes: Practical Examples of LCD Modeling and Optimization 91
  4.1 Optimization of LCD Performance in Various Electro-optical Modes 91
    4.1.1 Electrically Controlled Birefringence 91
    4.1.2 Twist Effect 101
    4.1.3 Supertwist Effect 109
    4.1.4 Optimization of Optical Performance of Reflective LCDs 116
  4.2 Transreflective LCDs 119
    4.2.1 Dual-Mode Single-Cell-Gap Approach 119
    4.2.2 Single-Mode Single-Cell-Gap Approach 122
  4.3 Total Internal Reflection Mode 124
  4.4 Ferroelectric LCDs 131
    4.4.1 Basic Physical Properties 131
    4.4.2 Electro-optical Effects in FLC Cells 135
  4.5 Birefringent Color Generation in Dichromatic Reflective FLCDs 145
    References 149

5 Necessary Mathematics. Radiometric Terms. Conventions. Various Stokes and Jones Vectors 153
  5.1 Some Definitions and Relations from Matrix Algebra 153
    5.1.1 General Definitions 153
    5.1.2 Some Important Properties of Matrix Products 160
    5.1.3 Unitary Matrices. Unimodular Unitary $2 \times 2$ Matrices. STU Matrices 160
    5.1.4 Norms of Vectors and Matrices 163
    5.1.5 Kronecker Product of Matrices 166
    5.1.6 Approximations 167
5.2 Some Radiometric Quantities. Conventions 167
5.3 Stokes Vectors of Plane Waves and Collimated Beams Propagating in Isotropic Nonabsorbing Media 169
5.4 Jones Vectors 171
5.4.1 Fitted-to-Electric-Field Jones Vectors and Fitted-to-Transverse-Component-of-Electric-Field Jones Vectors 171
5.4.2 Fitted-to-Irradiance Jones Vectors 172
5.4.3 Conventional Jones Vectors 175
References 176

6 Simple Models and Representations for Solving Optimization and Inverse Optical Problems. Real Optics of LC Cells and Useful Approximations 177
6.1 Polarization Transfer Factor of an Optical System 178
6.2 Optics of LC Cells in Terms of Polarization Transport Coefficients 182
6.2.1 Polarization-Dependent Losses and Depolarization. Unpolarized Transmittance 185
6.2.2 Rotations 187
6.2.3 Symmetry of the Sample 190
6.3 Retroreflection Geometry 192
6.4 Applications of Polarization Transport Coefficients in Optimization of LC Devices 195
6.5 Evaluation of Ultimate Characteristics of an LCD that can be Attained by Fitting the Compensation System. Modulation Efficiency of LC Layers 207
References 216

7 Some Physical Models and Mathematical Algorithms Used in Modeling the Optical Performance of LCDs 217
7.2 Transfer Matrix Technique and Adding Technique 237
7.2.1 Transfer Matrix Technique 238
7.2.2 Adding Technique 242
7.3 Optical Models of Some Elements of LCDs 246
References 248

8.1 General Properties of the Electromagnetic Field Induced by a Plane Monochromatic Wave in a Linear Stratified Medium 252
8.1.1 Maxwell’s Equations and Constitutive Relations 252
8.1.2 Plane Waves 256
8.1.3 Field Geometry 259
8.2 Transmission and Reflection Operators of Fragments (TR Units) of a Stratified Medium and Their Calculation 275
8.2.1 EW Jones Vector. EW Jones Matrices. Transmission and Reflection Operators 275
8.2.2 Calculation of Overall Transmission and Overall Reflection Operators for Layered Systems by Using Transfer Matrices 281
## Contents

8.3 Berreman’s Method 283  
8.3.1 Transfer Matrices 283  
8.3.2 Transfer Matrix of a Homogeneous Layer 285  
8.3.3 Transfer Matrix of a Smoothly Inhomogeneous Layer. Staircase Approximation 287  
8.3.4 Coordinate Systems 289  
8.4 Simplifications, Useful Relations, and Advanced Techniques 291  
8.4.1 Orthogonality Relations and Other Useful Relations for Eigenwave Bases 291  
8.4.2 Simple General Formulas for Transmission Operators of Interfaces 297  
8.4.3 Calculation of Transmission and Reflection Operators of Layered Systems by Using the Adding Technique 303  
8.5 Transmissivities and Reflectivities 304  
8.6 Mathematical Properties of Transfer Matrices and Transmission and Reflection EW Jones Matrices of Lossless Media and Reciprocal Media 311  
8.6.1 Properties of Matrix Operators for Nonabsorbing Regions 311  
8.6.2 Properties of Matrix Operators for Reciprocal Regions 313  
8.7 Calculation of EW 4 × 4 Transfer Matrices for LC Layers 319  
8.8 Transformation of the Elements of EW Jones Vectors and EW Jones Matrices Under Changes of Eigenwave Bases 322  
8.8.1 Coordinates of the EW Jones Vector of a Wave Field in Different Eigenwave Bases 322  
8.8.2 EW Jones Operators in Different Eigenwave Bases 326  
References 328  

9 Choice of Eigenwave Bases for Isotropic, Uniaxial, and Biaxial Media 331  
9.1 General Aspects of EWB Specification. EWB-generating routines 331  
9.2 Isotropic Media 338  
9.3 Uniaxial Media 342  
9.4 Biaxial Media 352  
References 365  

10 Efficient Methods for Calculating Optical Characteristics of Layered Systems for Quasimonochromatic Incident Light. Main Routines of LMOPTICS Library 367  
10.1 EW Stokes Vectors and EW Mueller Matrices 368  
10.2 Calculation of the EW Mueller Matrices of the Overall Transmission and Reflection of a System Consisting of “Thin” and “Thick” Layers 375  
10.3 Main Routines of LMOPTICS 384  
10.3.1 Routines for Computing 4 × 4 Transfer Matrices and EW Jones Matrices 384  
10.3.2 Routines for Computing EW Mueller Matrices 388  
10.3.3 Other Useful Routines 391  
References 392  

11 Calculation of Transmission Characteristics of Inhomogeneous Liquid Crystal Layers with Negligible Bulk Reflection 393  
11.1 Application of Jones Matrix Methods to Inhomogeneous LC Layers 394  
11.1.1 Calculation of Transmission Jones Matrices of LC Layers Using the Classical Jones Calculus 394  
11.1.2 Extended Jones Matrix Methods 404  
11.2 NBRA. Basic Differential Equations 409  
11.3 NBRA. Numerical Methods 420  
11.3.1 Approximating Multilayer Method 421  
References 425
Contents

11.3.2 Discretization Method 427
11.3.3 Power Series Method 428
11.4 NBRA. Analytical Solutions 430
11.4.1 Twisted Structures 430
11.4.2 Nontwisted Structures 432
11.4.3 NBRA and GOA. Adiabatic and Quasiadiabatic Approximations 434
11.5 Effect of Errors in Values of the Transmission Matrix of the LC Layer on the Accuracy of Modeling the Transmittance of the LCD Panel 437
References 438

12 Some Approximate Representations in EW Jones Matrix Method and Their Application in Solving Optimization and Inverse Problems for LCDs 441
12.1 Theory of STUM Approximation 442
12.2 Exact and Approximate Expressions for Transmission Operators of Interfaces at Normal Incidence 447
12.3 Polarization Jones Matrix of an Inhomogeneous Nonabsorbing Anisotropic Layer with Negligible Bulk Reflection at Normal Incidence. Simple Representations of Polarization Matrices of LC Layers at Normal Incidence 463
12.4 Immersion Model of the Polarization-Converting System of an LCD 466
12.5 Determining Configurational and Optical Parameters of LC Layers With a Twisted Structure: Spectral Fitting Method 474
12.5.1 How to Bring Together the Experiment and Unitary Approximation 476
12.5.2 Parameterization and Solving the Inverse Problem 480
12.5.3 Appendix to Section 12.5 489
12.6 Optimization of Compensation Systems for Enhancement of Viewing Angle Performance of LCDs 490
References 504

13 A Few Words About Modeling of Fine-Structure LCDs and the Direct Ray Approximation 507
13.1 Virtual Microscope 508
13.2 Directional Illumination and Diffuse Illumination 513
References 516

A LCD Modeling Software MOUSE-LCD Used for the HKUST Students Final Year Projects (FYP) from 2003 to 2011 517
A.1 Introductory Remarks 517
A.2 Fast LCD
A.2.1 TN Cell 517
A.2.2 Effect of d/p Ratio 519
A.2.3 Effect of K_{22}/K_{11} 520
A.2.4 Effect of K_{33}/K_{11} 520
A.2.5 Effect of Δε 521
A.2.6 Effect of γ_{1} 521
A.2.7 Effect of Anchoring Strength W 523
A.2.8 Optimized TN Cell With Fast Response Time 523
A.2.9 Other LC Modes 524
A.3 Color LCD
A.3.1 The Super-Twisted Nematic Cell 524
A.3.2 STN Birefringent Colors in Transmissive and Reflective Modes 525
A.4 Transflective LCD
   A.4.1 Vertical Aligned Nematic Cell 525
A.5 Switchable Viewing Angle LCD 535
A.6 Optimal e-paper Configurations 535
A.7 Color Filter Optimization 536
   References 536

B Some Derivations and Examples 537
B.1 Conservation Law for Energy Flux 537
B.2 Lorentz’s Lemma 538
B.3 Nonexponential Waves 538
B.4 To the Power Series Method (Section 11.3.3) 540
B.5 One of the Ways to Obtain the Explicit Expressions for Transmission Jones Matrices of an Ideal Twisted LC Layer 541
   Reference 543

Index 545
Series Editor’s Foreword

Liquid crystal displays are the bedrock of the flat panel display industry. Their success and their continued improvement in all aspects of performance are due in substantial part to improvements in the fundamental understanding of how liquid crystal structures interact with forces applied by external electrical fields and by the intrinsic potential differences which exist at boundaries between dissimilar materials.

Several computer modelling systems are commercially available. They enable users to predict the properties of displays, avoiding the necessity to test every new idea by experiment. They are essentially black boxes into which are inputted the properties of materials, cell dimensions, applied voltage and other data, and which output the optical properties of a display as functions of time, applied voltage, wavelength and viewing angle. Their use requires no fundamental understanding of the thermodynamics or mechanics of liquid crystal (LC) interactions and therein lies a potential problem. For reasons of efficiency and minimising computer time, most, if not all modelling routines operate on simplified and approximated formulæ. Under some circumstances these approximations can lead to unforeseen errors and this is a topic which is addressed in unprecedented detail in this volume. But first it contains an exposition of the fundamentals from a description of polarized light through the calculation of its interaction with LC layers by the Jones calculus to predict the properties of cell structures. Next are presented worked examples of different transmissive and reflective nematic and ferroelectric modes using modelling software developed by the authors. The second part of the book provides a more detailed analysis of mathematical methods, starting from the basic mathematics and matrix algebra specific to LC modelling. It then progresses from describing relatively simple models to a description of rigorous electromagnetic methods to describe the optics of 1D inhomogeneous media and their use for numerical modelling of LC optics. The impact of approximations on computational accuracy is discussed throughout. The final chapter of the book touches on layers which are anisotropic in two dimensions, an important topic for LCDs which increasingly use multi-domain pixel structures.

The detailed contents of each chapter are described by the authors in their introduction, but my purpose in presenting this briefer description here is to show what a comprehensive book this is. It goes even further because a companion website http://www.wiley.com/go/yakovlev/modelinglcd contains the well commented source code of the program library LMOPTICS, which is a collection of routines for calculating the optical characteristics of multilayer systems, based on the methods described in this book. It also contains a set of sample programs which exemplify the application of this library and the methods described in this book to modelling LCDs.

This book and its companion website provide a comprehensive operational base for scientists and engineers who wish to make reliable modelling experiments. It provides a wealth of information for academic researchers and students engaged in condensed matter physics which is of relevance not just to displays but to LC-based photonic devices in general.

Anthony Lowe
Braishfield, UK, 2014