ALLADI RAMAKRISHNAN (1923-2008): Picture taken in Madras, India in 1958 upon his return from the Institute for Advanced Study in Princeton when he was full of visions to create a similar center for advanced learning in Madras
The Legacy of Alladi Ramakrishnan in the Mathematical Sciences
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Preface

Alladi Ramakrishnan (1923–2008) was an eminent scientist who had a wide range of research interests in theoretical and mathematical physics. Professor Ramakrishnan made significant contributions to probability and statistics, elementary particle physics, cosmic rays and astrophysics, matrix theory, and the special theory of relativity. Ramakrishnan believed strongly that in addition to doing fundamental research, one must contribute to the advancement of the profession. Inspired by his visit to the Institute for Advanced Study in Princeton in 1957–1958, he returned to Madras and began the Theoretical Physics Seminar at his family home Ekamra Nivas. These seminars were ultimately responsible for the creation of MATSCIENCE, The Institute of Mathematical Sciences in 1962. This institute, of which he was the Director for its first 21 years, has grown steadily in size and stature, and is his monumental contribution to the profession. In a distinguished scientific life that has spanned more than five decades, Professor Ramakrishnan has come into close contact with, and was influenced by, several eminent mathematicians and physicists, and has moulded the careers of his several students and young researchers. This volume, which is a tribute to his great legacy, not only deals with his significant contributions to research and the profession, but also contains a fine collection of research and survey papers by leading physicists and mathematicians that cover a broad range of areas in the mathematical sciences.

The first part of this volume is about Professor Alladi Ramakrishnan and his contributions. The book begins with an article entitled “Contributions of Alladi Ramakrishnan to the Mathematical Sciences” in which the remarkable career and contributions of Ramakrishnan are described by his son Krishnaswami Alladi who was very close to his father and accompanied Professor Ramakrishnan regularly on his worldwide scientific trips. Included in Krishna’s article is a description of Ramakrishnan’s visit to the Institute for Advanced Study in Princeton in 1957–1958, and the subsequent exciting series of events in Madras which led to the creation of MATSCIENCE. This is immediately followed by an article on Alladi Ramakrishnan’s (now famous) Theoretical Physics Seminar. The list of eminent speakers at the seminar and the list of students who attended are provided.

The creation of MATSCIENCE was heralded with enthusiasm by scientists around the world. Telegrams and letters came pouring in for the inauguration. A few sample congratulatory telegrams from world famous physicists and mathematicians
are reproduced and brief comments are made about the person sending the telegram or letter and Ramakrishnan’s association with that scientist.

The creation of MATSCIENCE was like a dream come true! The next item in this volume is Alladi Ramakrishnan’s speech *The Miracle has Happened* which he gave at the inauguration of MATSCIENCE on 3 January 1962. In his inimitable style, Professor Ramakrishnan describes the series of incredible events, each as improbable as the other, that took place in rapid succession. Ramakrishnan was charged with emotion as he gave this most inspiring speech, which is actually a model in English diction!

Professor Ramakrishnan believed in maintaining close contact with the international scientific community. Just as he invited eminent scientists regularly to the Theoretical Physics Seminar and to MATSCIENCE, he traveled across the globe annually to disseminate the work of his group. In these travels, he made new contacts and that invigorated not only his own research, but also the visiting scientists program at MATSCIENCE. Thus, we have included a brief description of some of Ramakrishnan’s significant overseas trips.

Part I of the volume concludes with the list of scientific publications of Professor Alladi Ramakrishnan, and the list of his Ph.D. students.

Parts II–IV of the volume constitute research and survey papers by physicists and mathematicians who got to know Professor Alladi Ramakrishnan very well over the years. The range of topics covered by these papers is broad as were the research interests of Professor Ramakrishnan. The papers have been grouped as follows – Part II: pure mathematics, Part III: probability and statistics, and Part IV: applied mathematics and theoretical physics. Some of Professor Ramakrishnan’s former Ph.D. students and grand students have contributed papers included in Parts III and IV. Within each of the parts of the volume, the papers are listed alphabetically by author’s names.

**Part II: Pure Mathematics**

Shreeram Abhyankar, a leading algebraic geometer, admired Ramakrishnan not only for his research, but also for creating an institute for advanced study in the mathematical sciences in India. Abhyankar, who takes great pride in India’s intellectual past, himself created and directed a mathematics institute in Pune, Maharashtra, called the *Bhaskaracharya Prathistama*. In a massive paper jointly dedicated to Professor Ramakrishnan and his father, Abhyankar discusses extensions of his important work of 1967 on “gap invariance” with the intention of applying these ideas to a famous unsolved problem in algebraic geometry, namely, the *Jacobian Conjecture*.

Alladi Ramakrishnan was very much interested in using combinatorics to provide elegant proofs of identities and to use combinatorial insight to obtain generalizations and extensions. In this spirit, Krishna Alladi studies partitions into nonrepeating odd parts in a novel combinatorial way using 2-modular Ferrers graphs and their under-
lying Durfee squares to provide a unified treatment of several important identities in the theory of partitions and $q$-series.

Professor Ramakrishnan was fascinated by the symmetries and properties of the Pascal triangle which he often used to explain various enumeration problems arising in the theory of probability. Catalan numbers, which are defined using the middle binomial coefficients of the Pascal triangle, arise in a variety of settings. In a charming article, George Andrews investigates a $q$-analogue of the Catalan numbers and establishes several identities for these $q$-analogues, from which classical identities for the Catalan numbers fall out as special cases.

Another lifelong passion for Ramakrishnan was Euclidean geometry which he used to explain difficult concepts in the theory of special relativity. Richard Askey’s paper deals with the beautiful theorem of Ptolemy on cyclic quadrilaterals and the extension of this result by the Indian mathematician Brahmagupta.

Alladi Ramakrishnan was also very proud of India’s cultural and intellectual heritage. Naturally he was a great admirer of Ramanujan. In a joint paper with his student Atul Dixit, Bruce Berndt, one of the greatest authorities on Ramanujan’s work, discusses a transformation formula of Ramanujan and how this leads to transformations involving the Gamma and Riemann zeta functions. This transformation formula of Ramanujan may be found in the book “Ramanujan’s Lost Notebook and other unpublished papers” that was released during the Ramanujan Centennial in 1987. But this particular transformation formula is not in Ramanujan’s lost notebook discovered by George Andrews in 1976 at the Wren Library in Cambridge University, but is in the “loose papers” that were located in Oxford University Library.

The area of quadratic forms has witnessed dramatic progress in the last few years including the resolution of a problem on universal quadratic forms stemming from Ramanujan. Alexander Berkovich and William Jagy show how certain modular identities of degree 3 discovered by Ramanujan can be used to establish some very appealing positivity results for some integral ternary quadratic forms.

Asking questions of an additive nature for integers defined multiplicatively leads to very intriguing problems. Jean-Marc Deshouillers and Florian Luca, leading authorities in additive number theory, discuss the frequency of integers for which $n^2$ is a sum of three squares, and show that the density of such integers is at least $7/8$. This result is extremely interesting in the light of the classical theorem of Lagrange which asserts that every positive integer is a sum of (at most) four squares, and the simple observation that integers of the form $8k + 7$ cannot be represented as a sum of three squares.

Alladi Ramakrishnan was a great admirer of Euler for his many fundamental contributions. He once wrote an article on the charms of Euler’s $e$. So it is only appropriate that there is a paper in this volume emphasizing Euler’s work. Dominique Foata’s paper “Eulerian polynomials: from Euler’s time to the present” provides a beautiful survey of the topic. Foata starts with Euler’s memoir of 1755 to find out Euler’s motivation to study these polynomials. He then describes how these polynomials emerged in a $q$-generalized form in the work of Carlitz in the twentieth century and describes the underlying combinatorics. The contents of this paper were
delivered by Professor Foata in the Tenth Ulam Colloquium at the University of Florida in February 2008 and Professor Alladi Ramakrishnan attended that lecture.

The interaction between number theory and physics has attracted a lot of attention in recent years. Continuing his earlier investigations on connections between the Epstein zeta function and crystal symmetries, Shigeru Kanemitsu in joint work with Haruo Tsukada discusses several interesting examples, showing how crystal symmetry may be understood via zeta symmetry.

One of the main conjectures in the theory of linear forms is due to Minkowski on products of linear forms. Minkowski’s conjecture has been proved for six dimensions or less, but the general result is still unproven. In his paper, Raghavan treats a modified problem in a novel fashion and obtains similar results to Minkowski’s conjecture.

The penultimate paper of Part II is the seminal work of Peter Sin and John Thompson on the divisor matrix, Dirichlet series, and $SL(2, \mathbb{Z})$. Although divisors of integers have been studied since antiquity, no one has done a systematic study of the infinite upper triangular matrix $[a_{i,j}]$, where $a_{i,j} = 1$ if $i$ divides $j$ and 0 otherwise. Thompson and Sin explore connections between this matrix, Dirichlet series, and $SL(2, \mathbb{Z})$. The subject matter of this paper was Professor Thompson’s talk in Oslo in May 2008 after he received the Abel Prize. We are honored that this fundamental paper is included in this volume.

The final article in Part II is a letter by Michel Waldschmidt in which he proves a conjecture of Alladi Ramakrishnan on circulants. Professor Ramakrishnan was intrigued by the Lorentz Transformation in Special Relativity and provided new and elegant derivations of it. He wrote a paper “Pythagoras to Lorentz via Fermat” in which instead of considering the Fermat equation as the generalization of the Pythagorean equation, he studied an $n$-dimensional circulant generalization of the Pythagorean equation. Alladi Ramakrishnan connected this to the Lorentz transformation and determined its rational solutions. In this context, he made a conjecture regarding circulants and the proof of this conjecture is provided by Michel Waldschmidt.

Part III: Probability and Statistics

Alladi Ramakrishnan did fundamental work in the theory of probability. Thus, it is appropriate that this volume contains excellent papers in probability and statistics.

Alladi Ramakrishnan, along with Homi Bhabha, made pioneering contributions to the theory of nuclear cascades by the use of stochastic processes. The opening paper of Part II by Krishna Athreya deals with the Galton–Watson branching processes and the associated branching random walk. The limiting behavior of the spatial distribution of points in certain point processes is investigated and an application to the photon–electron cascade is described.
The next paper by Malay Ghosh, Kwok Pui Choi, and Jialiang Li provides a smooth treatment of the logistic distribution without the use of contour integration. The authors show how to calculate the moments, the moment generating function, and the characteristic function.

The paper by C.R. Rao is a comprehensive review of entropy and cross-entropy. He discusses their characterizations and indicates possible applications. Entropy has been used in characterizing probability distributions in theoretical physics to which Professor Ramakrishnan has made fundamental contributions. Entropy has been used as a measure of diversity in environmental studies. Cross-entropy has emerged as a useful tool in solving stochastic and nonstochastic optimization problems.

The father and son team of Jayaram Sethuraman and Sunder Sethuraman discuss connections between Bernoulli strings and random permutations. In this regard, they point out very elegantly the connection between marked Poisson processes and Bernoulli strings.

Professor Ramakrishnan’s grand student P.R. Vittal, S. Jaisankar, and V. Muralidhar investigate storage models. Storage theory has received considerable attention, and two of the leading contributors to this field are Joe Gani and Pap Moran, contemporaries of Alladi Ramakrishnan. Vittal, Jaisankar, and Muralidhar discuss storage problems for a class of one-dimensional master equations with separable kernels.

The problem of testing equality of survival distributions has received considerable attention. In the final paper of Part III, S.S. Wu, P.V. Rao, and Aparna Raychaudhry address this problem on the basis of paired censored survival data. They utilize test statistics that consist of linear combinations of two appropriately chosen statistics. In addition, they present a method for estimating optimal weights for such linear combinations.

Part IV: Theoretical Physics and Applied Mathematics

Professor Alladi Ramakrishnan worked and directed students in several areas of theoretical physics and applied mathematics and the breadth of his interests is reflected in the topics covered by the authors of this section.

Imaging science has become one of the most active areas of research owing to applications ranging from determining the size of tumors to detection of tanks under foliage. Yunmei Chen, an authority in imaging science, and her student Xiaojing Ye present a novel variational model for inverse consistent deformable image registration. Their model is formulated as an energy minimization model and experimental results indicate the efficiency of their model.

Alladi Ramakrishnan’s former Ph.D. student V. Devanathan (who later became the head of the nuclear physics department at the University of Madras) discusses a statistical model for the quark structure of the nucleon in a joint paper with his Ph.D. student S. Karthiyyinini. Their paper contains a good description of both the static
and dynamic properties of the nucleon. A thermodynamic bag model is proposed to obtain realistic distribution functions that correctly yield the nucleon structure functions.

Alladi Ramakrishnan produced a technique called the \( \sigma \)-operation to construct the \( 4 \times 4 \) Dirac matrices from the \( 2 \times 2 \) anticommuting Pauli matrices. This led him to study the more general \( \omega \)-commutation (\( \omega \) is a root of unity) and the hierarchy of matrices satisfying the \( \omega \)-commutation. That was the evolution of Ramakrishnan’s \( L \)-matrix theory which he pursued in depth by himself and with his Ph.D. students. R. Jagannathan, a former Ph.D. student of Alladi Ramakrishnan who later became a professor at MATSCIENCE, provides a nice review of generalized Clifford algebras and their applications to physics. In doing so, he discusses various ramifications of the work of Alladi Ramakrishnan and his group and the extensions that he himself has obtained. The fact that generalized Clifford algebras are so pervasive is well brought out in this paper.

Finding \( q \)-analogues of classical functions and identities has proved to be extremely fruitful because the scope of applications is considerably broadened with the introduction of \( q \)-analogues. R. Jagannathan and R. Sridhar, former student and grand student of Alladi Ramakrishnan who later became professors at MATSCIENCE, discuss a \((p, q)\)-analog of the Rogers-Szegő polynomial and the \((p, q)\) oscillator in physics. Just as the Rogers-Szegő polynomial is associated with the \( q \)-oscillator algebra, the authors show that the \((p, q)\)-Rogers-Szegő polynomial is associated with the \((p, q)\)-oscillator algebra.

John Klauder’s paper “Rethinking renormalization” is a critical re-examination of the notion of renormalizability for several extreme types of quantum field theory. Normally, counter terms that are needed to remove divergences which arise in quantum field-theoretic calculations are introduced on a term-by-term basis after evaluation of suitable functional integrals. Klauder’s approach differs by excising divergence-causing terms in the integrand of functional integrals, thereby eliminating divergences altogether. Ultimately, the aim is to apply this technique to the difficult task of quantizing the gravitational field.

The father and daughter team of A.N. Mitra and Gargi-Mitra Delmotte present a rather broad description of pattern formation in crystals and crystal-like structures under the influence of magnetic fields. The ability of these structures for self-replication, compartmental organization, and fractionalization serves as a basis for theoretical speculations that organic life may have originated utilizing similar mechanisms.

The final paper in the volume is by R. Parthasarathy, a grand student of Alladi Ramakrishnan who later became a professor at MATSCIENCE. This is a review of the work of the Ehrenfest theorem in Abelian and non-Abelian quantum field theories. The theorem is shown to be valid in appropriately defined physical subspaces.

This volume which contains a fine collection of papers covering a broad range of topics in number theory, algebra, geometry, probability, statistics, theoretical, nuclear, and mathematical physics, and certain topics in applied mathematics is a fitting tribute to the memory of Alladi Ramakrishnan who had such a profound influence on the scientific profession. The contributors include some of his students
and grand students who themselves went on to pursue highly successful academic careers, and eminent mathematicians, physicists, probabilists, and statisticians, who got to know Professor Ramakrishnan and his work very well over the years. Our thanks to all the contributors of this volume. A special thanks to Professor Frank Garvan of the University of Florida who provided crucial help in assembling the TeX files of the papers for production. Felix Portnoy of Springer, New York, and Ejaz Ahmad in Chennai, India, did a fine job in typesetting the entire volume. Finally we wish to express our appreciation to Elizabeth Loew, Ann Kostant, Joachim Heinze and Hans Koelsch of Springer for their interest in producing this volume and their support throughout this venture.

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